Multitarget Classification and Estimation Using Clustering Techniques

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A modular approach to solving the multisensor-multitarget problem in space is presented. This approach combines the clustering techniques developed in the field of pattern recognition with the filtering techniques developed to estimate the states of several targets in space. Results from the clustering analysis are presented, and their relevance to the identification of targets and data fusion are described using two scenarios having multiple targets and using orbiting platforms as sensors. The estimation error histories of positions and velocities are also given.

I. Introduction

THE ability to detect, classify, and predict the motion of a cluster of unidentified targets is a crucial element in many current command, control, communication, and intelligence (C^3I) applications.[1,2] In the defense environment, there can be multiple targets whose states are to be estimated using observations from distributed sensor configurations. The detection-estimation system must address the problems of using the multitarget, multisensor data to trace a set of targets and the observations must determine which targets are present in the observation set. For example, a sensor might detect several targets n, while another sensor will detect only a subset m of the targets. The detection system must determine if the m targets are included in the n-target set, or whether there are really $n+m$ targets. Hence, in the multitarget-multisensor environment, the uncertainties associated with the measurements will include not only sensor error, but uncertainties related to the target identity as well. These uncertainties will involve the target configuration at the time of the measurements, which may contain actual targets, decoys, and/or specific measurement disturbances.

Consequently, the detection and separation of targets from decoys, real data from spurious data, and the proper identification of data sequence corresponding to a target become the first and foremost problem in the multitarget tracking application. Second, the data from the different sensors must becombined into a single batch and then separated into the data sequences corresponding to the various targets. Although the data from each individual sensor can be classified into clusters, it is not known a priori how to relate them to the clusters containing the corresponding target data collected by the other sensors. Third, the accurate estimation of the states of the target (e.g., position, velocity, and acceleration) must be accomplished in a rapidly changing environment. Both the dynamic models and the measurement functions are nonlinear, and therefore accurate determination of the states of the target in a near-real time computation is difficult to accomplish.

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II. Existing Methods in the Literature

Because of the complexity of the problems that appear in the various aspects of the multitarget tracking as outlined in Sec. I, considerable research has been undertaken to obtain improved solutions to these problems.[3,4] The literature on this research can be classified broadly into methods dealing with multisensor and multitarget (or single-target) tracking and methods for single-sensor and single-target tracking.

Multisensor-Multi- (or Single-) Target Tracking Likelihood Function Methods

Sittler[5] considered the problem of data association with reference to the surveillance problem. Additional target tracks were added whenever more than one observation was detected in the neighborhood of the predicted measurement, and a likelihood function to drop tracks that fell below a preselected detection threshold was used. A conceptually similar approach has been used by Smith and Buechler[6] for tracking multiple objects. Their branching algorithm consisted of a bank of parallel filters using the Kalman methodology. They used the "innovations" of the observations or observation residuals and the state estimates to make decisions about the measurement-track correlation and to combine the tracks. Porter and Englar[7] have solved the multitargeting problem, where the branching of tracks was based on the generalized likelihood. The drawback of the approach used in Refs. 5–7 lies in the fact that the computational and memory requirements can become large as the number of tracks under consideration increases.

Morefield[8] has used a maximum likelihood function to formulate a batch-processing technique for the tracking problem. He devised an algorithm to solve for the most likely data-association hypothesis and applied the theories of linear integer programming to find the most likely set of tracks. These tracks were used with single-target tracking methods to estimate the states of each target. A drawback of all of these hypothesis-oriented and maximum likelihood techniques, as pointed out by Bar-Shalom,[3] is that the estimates and the associated covariances do not account for the measurement-origin uncertainty.

Probabilistic Data, Association Filters

The probabilistic data-association filters are based on a scheme where the posteriori association of probabilities is computed for all the current candidate measurements in a validation gate and used to form a weighted sum of residuals for updating the target's state using an appropriate filter. Singer et al.[9] formulated a general probabilistic data-association filter wherein the decomposition of the state estimate was done in terms of all combinations of measurements from initial to most recent time.

As opposed to the global concept of Singer et al.,[9] Bar-Shalom and Tse[10] have used only the latest set of measurements in the formulating a probabilistic data-association filter. Their filter, which does not permit track splitting, was developed for a single target and allows for clutter. With a joint probability data-association method, Fortmann et al.[11] extended the single-target formulation in Ref. 10 to the multitargeting problem. The algorithm of Fortmann et al. was applied to a passive sonar-tracking problem in Ref. 11 with multiple sensors and targets. Their simulation results showed that the joint probability association method tracks two interfering targets better than the track-splitting maximum likelihood method. It should be noted that the probabilistic data-association related papers[9-11] do not discuss track initiation, which is an important element in solving the multitarget problems because of the high level of uncertainty in the initial states.

Reid[12] has devised a probability data-association method to solve the multitarget-tracking problems. His approach is Bayesian and can handle several targets. Reid's method also has the capability for track initiation. There is, however, a problem of growing memory due to the increase in the number of hypotheses associated with the measurements.

The problem of correlation of measurements from several sensors belonging to the same targets has been dealt with by Chang and Youens[13] who showed that it is similar to the assignment problem in operations research. They have summarized an algorithm for multiple-sensor measurement correlation based on such an approach. The associated problems with such an approach or where it fits into any of the other tracking methods were not addressed.

III. Multitarget Estimation Using Clustering Analysis

As opposed to most of the methods in the multisensor-multitarget algorithms in the literature, the approach used in this investigation of the multitarget-tracking problem is modular (Fig. 1). All of the problems identified in Section I are handled separately. However, interactions between the different modules of tracking and identification are possible for mutual improvement of the assigned tasks. The approach of this study is based on a batch-processing algorithm,[14,15] but its output is used both for the initiation of target tracks and for more accurate identification and association of the measurement data. The state estimation is formulated as a recursive Kalman filter in which a batch-type calculation is used for the measurement incorporation.

First, by applying a hierarchical clustering technique,[16,17] the multitarget data is sorted into data clusters for each sensor. The clusters from different sensors for the same targets are properly matched with the results from a batch filter. After each sensor's data are separated into clusters for the respective targets, the related clusters are combined into single-target data sets. The multitarget-tracking problem is thus reduced to several single-target tracking problems. The states of the individual targets are then estimated with an extended Kalman filter.[14] This approach to solve the multitarget-tracking problem consists of the steps presented in Fig. 1 which will be described.

Simulated observations are generated for selected trajectories (step 1 in Fig. 1). The types of observations considered in this study are 1) relative range between the target and the observer; 2) the relative range rate; 3) the azimuth angle of the target from the sensor; and 4) the corresponding elevation angle. For each sensor, observations from different targets are merged to create the multitarget data set used for clustering analysis (11) (step 2 in Fig. 1).

Cluster analysis is a field that seeks to sort data points or objects into natural groups based upon their attributes. One of

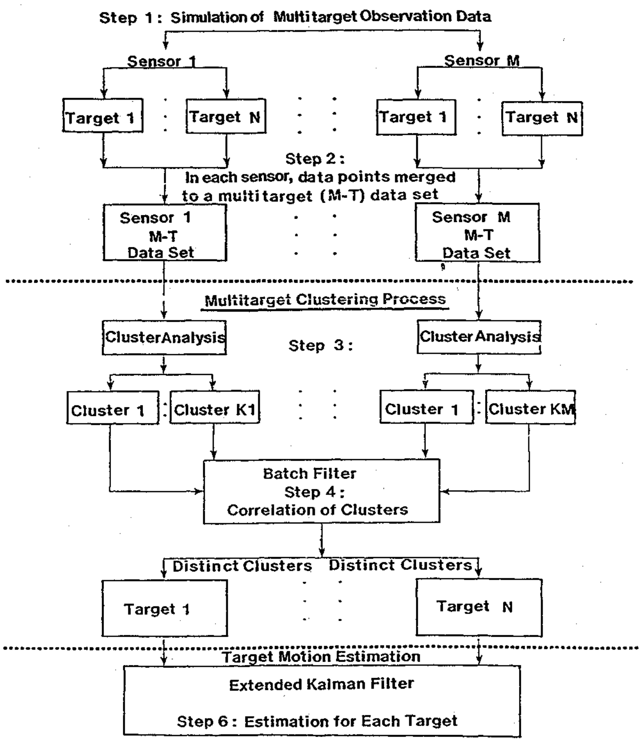

Fig. 1 Diagram for data flow.

its applications is to construct a category structure for the data points. No a priori functional form or conditional relationship is assumed between its data points and their attributes. The category structure shows the degree of similarity among the data points. In this paper, a hierarchical clustering method is used to sort the multitarget data set for each sensor (step 3 of Fig. 1). This method has the following features:

1) The simulated data consisting of time, range, range rate, azimuth, and elevation are used as attributes to describe a data point.

2) All five attributes are normalized so that the range of their values lies between 0 and 1. The underlying reason is to avoid the magnitudes of any one datum from dominating the analysis and reducing/eliminating information from other data being used in the process. It is also possible to weigh the actual data with the respective covariances in order to emphasize their relative importance, if needed.

3) The dissimilarity coefficient used to calculate the measures of dissimilarity between a pair of data points was the average Euclidean distance d_{jk}

$$d_{jk} = \left| \frac{1}{n} \sum_{i=1}^{n} (x_{ij} - x_{ik})^2 \right|^{1/2} \tag{1}$$

where x_{ij} is the ith attribute of the jth data point, n the number of attributes, and d_{jk} is the distance between the jth and kth data points. Then, the resemblance matrix (the dissimilarity matrix) based on this coefficient is calculated from the matrix of data attributes.

4) The single-linkage hierarchical clustering method[16] is used in this study. As an example of this clustering method, consider a sequence of partitions of 10 samples. At the first level, since all of the samples are at least slightly different (or dissimilar), there are 10 clusters, each cluster containing exactly one sample. At the next level of dissimilarity, there is a partition into nine clusters and then eight and so on, until there is one cluster of 10 samples at the lowest level of dissimilarity. If the sequence has the property that two samples in the same cluster at a dissimilarity level remain together at all lower levels, then the sequence is called a hierarchical clustering. In the single-

Table 1 Sensors for scenario 1[a]

	Initial conditions for sensor orbits		
	1	2	3
Inclination, deg	85.0000	85.0000	85.0000
Long Asc node, deg	62.0089	182.0098	302.0089
Eccentricity	0.3689374	0.3689374	0.3689374
Arg of perigee, deg	281.3211	281.3211	281.3211
Mean anomaly, deg	132.4823	132.4823	132.4823
Mean motion, rev/day	8.0653612	8.06536912	8.06536912

[a]Epoch: 16/04:25:07 UTC July 1986 (Day 197.18410880).

linkage method, at each stage, after two clusters 1 and 2 are merged to form cluster 3, the similarity between cluster 3 and any other cluster x is determined by defining the metric S_{ij} as

$$S_{3x} = \min(S_{1x}, S_{2x}) \tag{2}$$

The quantity S_{3x} is the distance between the two closest members of clusters 3 and x. If clusters 3 and x are merged, then for any sample in the resulting cluster, the distance to its nearest neighbor would be at most S_{3x}. This method is known as single linkage because clusters are joined at each level by the shortest link between them.

The single-linkage method is applied to generate tree-type diagrams. The tree diagram has many clustering levels at which data points and smaller clusters are joined. Consequently, all smaller clusters are linked into a single large cluster. The criterion for determining the number of targets in a multitarget problem is the number of significant jumps in the clustering levels in the tree diagram (done manually in the current application).

After the clustering technique is applied on the multitarget data set for each sensor, the clusters from different sensors for the same target are properly matched by the application of a batch filter on the pairwise combinations of clusters to determine the relationship between clusters (step 4 in Fig. 1). A correctly matched data set will produce low root-mean-square (rms) residuals of the initial states, and an incorrectly matched set will exhibit high rms residuals because of incompatible target dynamics. After applying a batch filter to different combinations of clusters of data, the best correlations of clusters can be achieved. Note that computational load in such a sensor-to-sensor correlation using batch process can grow as the product of the sensors and targets. With the clusters of data from different sensors, the batch algorithm is used to initiate the track of each target. Subsequently, the extended Kalman filter is used to estimate the tracks on-line.

IV. Simulation and Numerical Results

Simulation

The stochastic equations of motion for a particle in space are given in Appendix A.[18] In cases where the target is thrusting, this model can be easily extended to accommodate the powered phase.

The three sensors in this study are assumed to be orbiting platforms and their orbits are presented in Tables 1 and 2 for scenarios 1 and 2, respectively. The orbits have been so chosen as to assure observability of the targets during their entire flight. These orbits have an apogee altitude of 8000 km and a perigee altitude of 250 km. The measurements are the range from the sensor to the target (relative range), the range rate, the bearing angle (azimuth), and the elevation angle. Their relations to the target states are presented in Appendix B. The associated measurement variances are given in Table 3.

Table 3 Measurement covariances σ^2

Measurement type	Scenario 1	Scenario 2
Range	1 cm^2	1 m^2
Range-rate	1 cm^2/s^2	1 cm^2/s2
Azimuth	10^{-7} rad^2	10^{-7} rad^2
Elevation	10^{-7} rad^2	10^{-7} rad^2

Table 4 Launch and impact points

Launch points (LP)/ Impact point (IP)	X, km	Y, km	Z, km
LP1	1254	−3794	4790
LP2	669	−4001	4921
LP3	−198	−3542	5300
LP4	443	−3612	5237
LP5	1698	−3422	5107
IP1	1763	4502	4159
IP2	659	4708	4251

Numerical results from two representative cases are analyzed in this paper. The results presented are chosen to illustrate the effectiveness of 1) the clustering technique to discriminate the data from different targets and identify the number of targets; 2) the batch algorithm to estimate a good approximately accurate set of initial conditions; and 3) the ability of an extended Kalman filter to track the individual targets.

The coordinates of the launch points (LP) with respect to an inertial geocentered coordinate system are presented in Table 4. The impact positions (IP) of the target are also presented in Table 4. It should be noted that these impact positions are calculated out of the trajectories from LP1. Even though geographically the impact points are fixed, their position vectors in the trajectories from the other LP's will be different in an inertial frame due to the rotation of the Earth.

Clustering Analysis

The first scenario consists of a target T1 launched from LP1 reaching IP1 and a target T2 from LP2 reaching IP2. The tree diagram, an output from the clustering analysis, is given in Fig. 2. The inputs to the clustering technique are the merged data (range, range rate, and azimuth) corresponding to T1 and T2 collected by sensor 1 at 10 sample points at 5-s intervals. It is easy to see from Fig. 2 that there are two distinct clusters, thereby indicating the existence of two targets. Another outcome of the tree diagram is that it has sorted out (measurement vector being indicated by the numbers) the measurements corresponding to individual targets. Such groupings are also found manually by clustering the data from other sensors.

In this investigation, most of the tree diagrams have significant jumps as in the case presented in Fig. 2. However, such a

Table 2 Sensors for scenario 2[a]

	Initial conditions for sensor orbits			
	1	2	3	4
Inclination, deg	85.000	85.0000	85.0000	85.0000
Long Asc node, deg	70.0000	140.0000	210.0000	350.0000
Eccentricity	0.3689374	0.3689374	0.3689374	0.3689374
Arg of perigee, deg	281.3211	281.3211	281.3211	218.3211
Mean anomaly, deg	132.4823	132.4823	132.4823	132.4823
Mean motion, rev/day	8.06536912	8.06536912	8.06536912	8.06536912

[a]Epoch: 16/04:25:07 UTC July 1986 (Day 197.18410880).

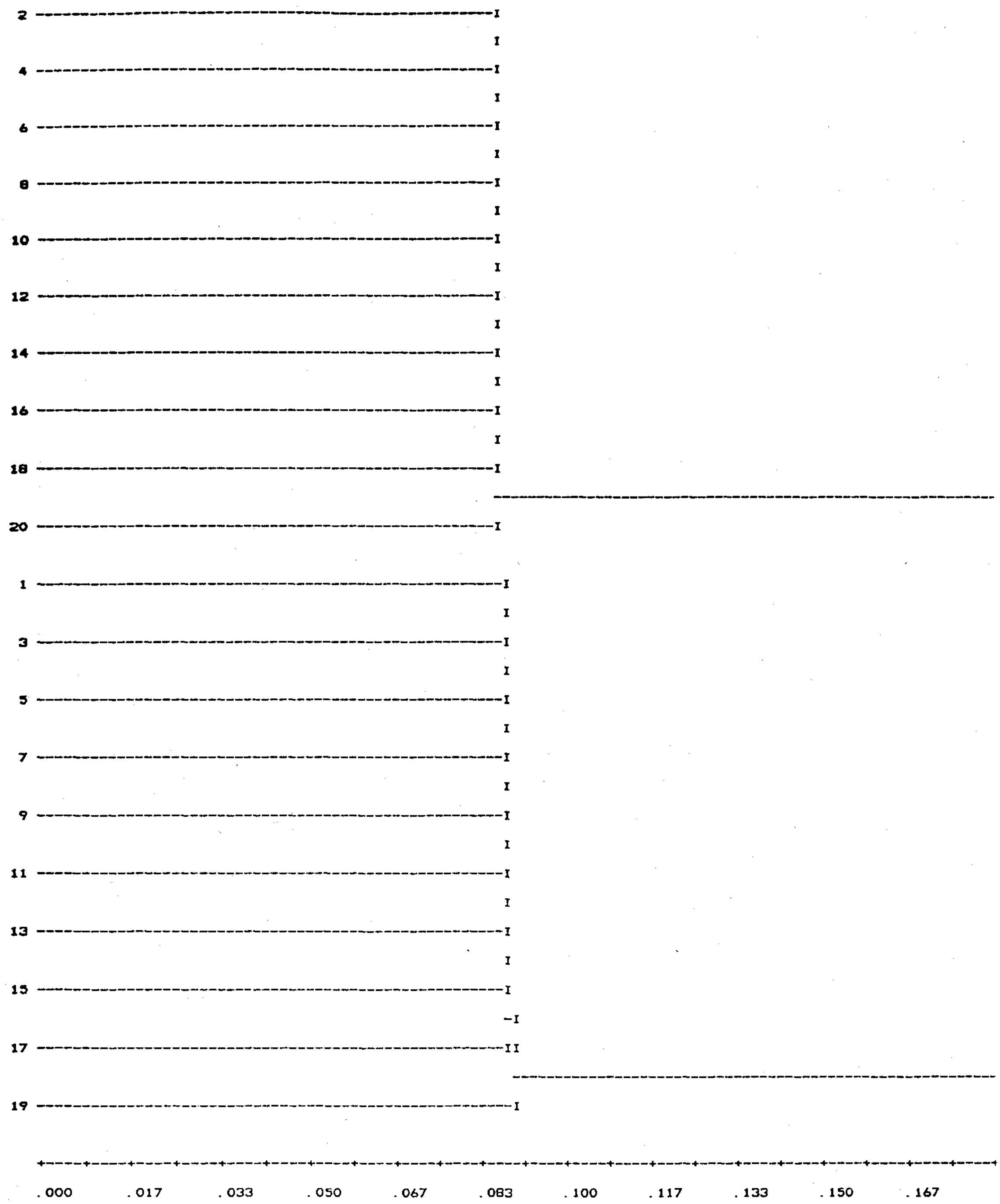

Fig. 2 Tree diagram from sensor 1 for scenario 1.

demarcation may not be present in application with inadequate measurements or unobservable geometry. An important requirement for the successful use of this technique is to study the tree diagrams carefully for different scenarios of the application and determine the proper threshold for successful and automated sorting of the clusters.

Intersensor Data Matching

In order to merge the data corresponding to each target from different sensors, each cluster of data from sensor 1 is merged with a cluster of data from sensor 2 and sensor 3, one at a time. With these as measurements, a batch filter is used to estimate the initial conditions iteratively. The convergence or divergence of the iterations on the initial conditions leads to the conclusion that the data groups from different clusters have emanated, respectively, from the same origin or different targets.

Estimation of Target States

With the data thus collected from all the sensors, the batch filter is used to generate initial conditions to be used with the extended Kalman filter. Initial error covariance of 10^8 in.^{2} and 10^4 m^2/s^2 are used for the positions and velocities, respectively. The off-diagonal elements are set to zero. The good convergence of position data and the sensitivity of the convergence to the available number of data points are presented in Table 5. With just 10 measurements on T1, the errors have been reduced in the y and z directions as much as 100 times. In other applications, more iterations may be needed.

With these initial conditions, the tracking of an extended Kalman filter is found to be very accurate. The results of the on-line estimation error, however, are presented for a more complex scenario. In this case, two targets from each of the 5 LP's are directed to both the IP's. The impact points are the same as before. The reason for picking this scenario is to test

Table 5 Initial condition estimation using batch filter

True initial conditions		
$X = 443$ km,	$Y = -3612$ km,	$Z = 5237$ km
Perturbations		
$\Delta X = 10$ km,	$\Delta Y =$ km,	$\Delta Z = 100$ km
Number of measurements		Errors after one iteration
10		$x_e = -543$ m
		$y_e = -953$ m
		$z_e = 775$ m
20		$x_e = -479$ m
		$y_e = -910$ m
		$z_e = 608$ m
30		$x_e = -396$ m
		$y_e = -396$ m
		$z_e = -445$ m

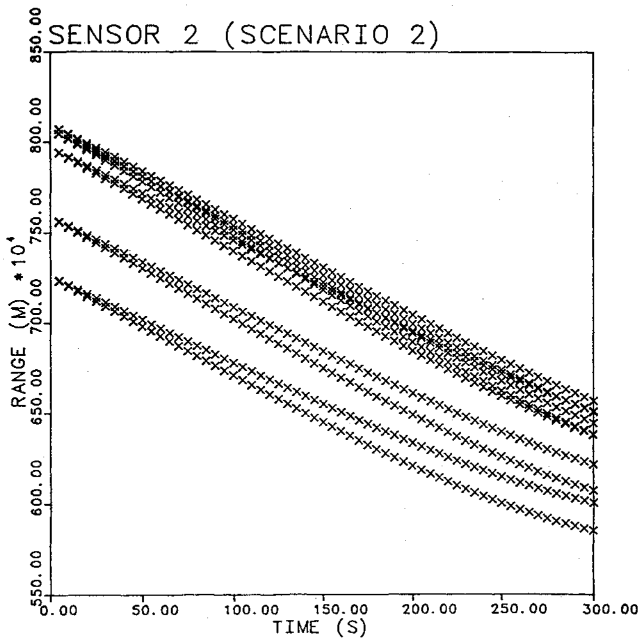

Fig. 3 Range measurements from sensor 2 for scenario 2.

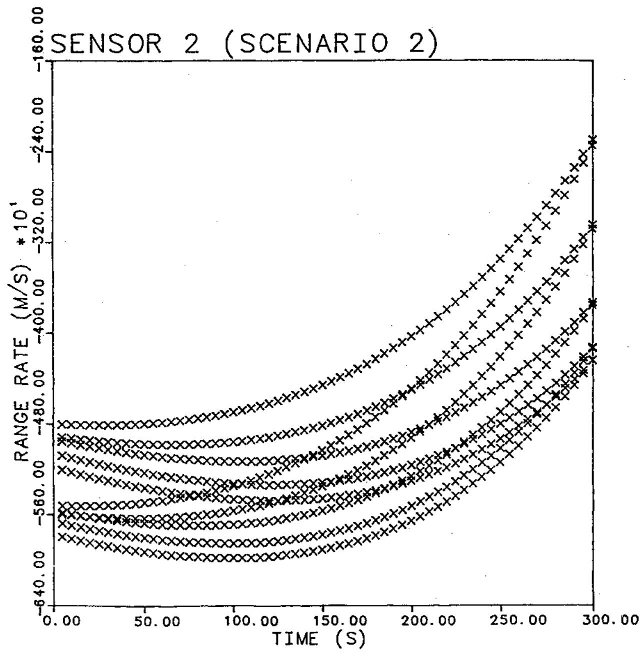

Fig. 4 Range-rate measurements from sensor 2 for scenario 2.

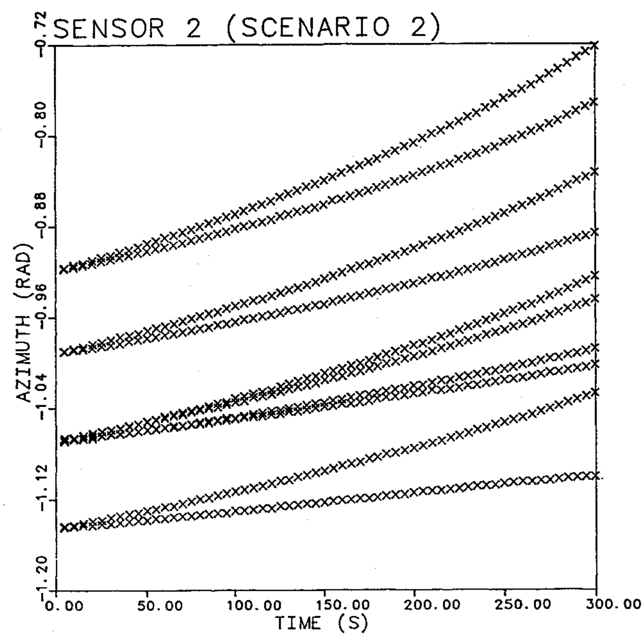

Fig. 5 Azimuth measurements from sensor 2 for scenario 2.

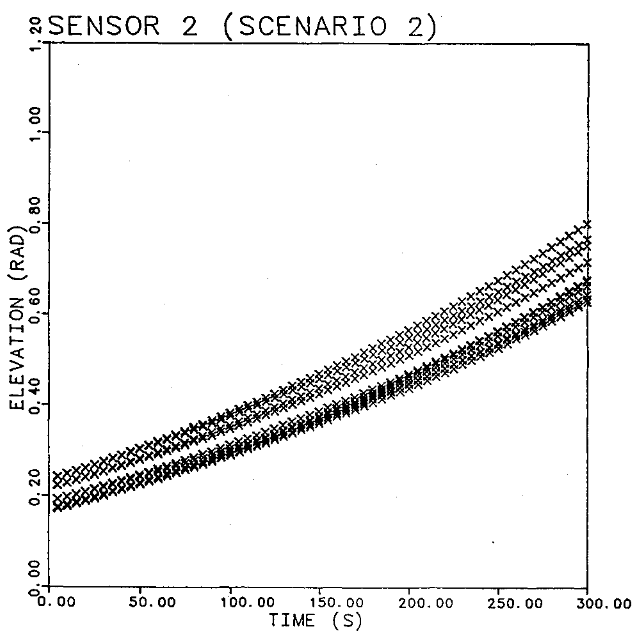

Fig. 6 Elevation measurements from sensor 2 for scenario 2.

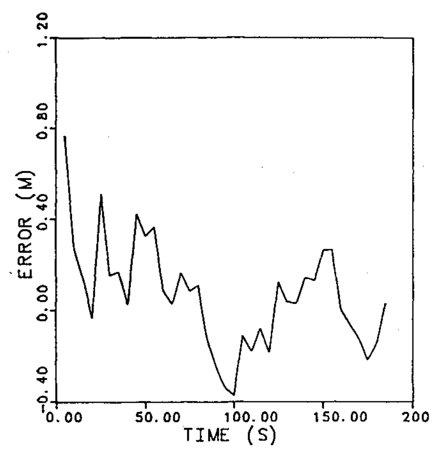

Fig. 7 Position error history in the radial direction (LP1 to IP1).

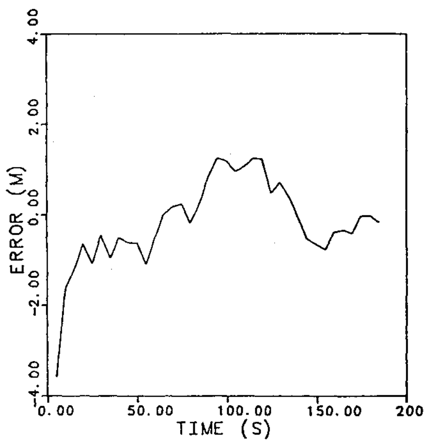

Fig. 8 Position error history in the along-track direction (LP1 to IP1).

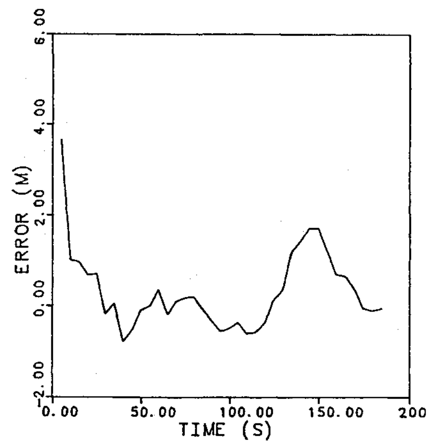

Fig. 9 Position error history in the cross-track direction (LP1 to IP1).

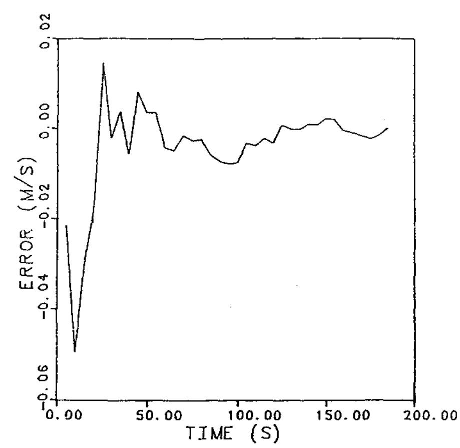

Fig. 10 Velocity error history in the radial direction (LP1 to IP1).

the effectiveness of separation of data by clustering analyses when a large number of targets and dense data are involved. The data corresponding to the second scenario are given in Figs. 3–6. It can be easily seen that there is extensive overlapping of measurements.

After the clusters are separated and merged, the initial conditions are found by the batch process as in the first scenario. With these as initial conditions, the extended Kalman filter is found to reduce the estimation errors remarkably. The errors in the estimated position components are presented in Figs. 7–9, and the errors in the velocity components are given in Figs. 10–12. These errors are calculated in the Cartesian coordinate frame and transformed to the radial, transverse, and

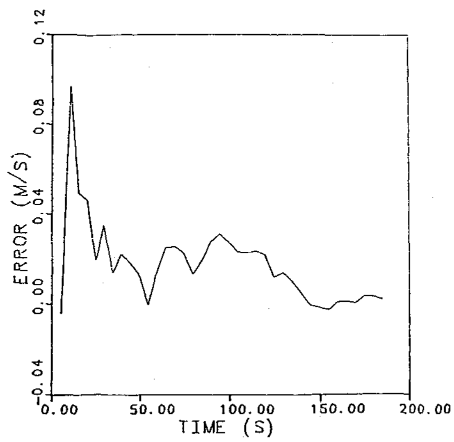

Fig. 11 Velocity error history in the along-track direction (LP1 to IP1).

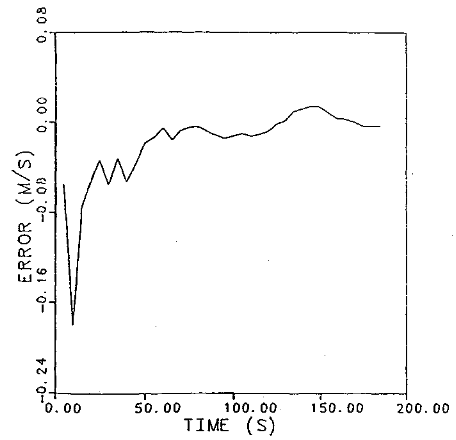

Fig. 12 Velocity error history in the cross-track direction (LP1 to IP1).

normal directions of the true orbital plane.

Even though the engagements last about 22 min, the estimation errors are presented only through 190 since the tracking is near perfect afterwards, and, therefore, not of interest.

In all of these cases, it is found to be sufficient to use only three measurement types, namely, the range, range rate, and the azimuth for the classification of data. It should be noted that since the purpose of this paper is to demonstrate the applicability of this modular approach to the multitarget problem, the Earth is taken as a spherical rotating body and other effects are ignored.

V. Conclusions

A modular approach has been presented as a method to solve a multitarget, multisensor problem at a centralized processor. This approach splits a multitarget problem into several single-target problems. Numerical results from a complex three-dimensional problem with nonlinear measurements have been given, and they show the effectiveness of this approach. Although the results in this study indicate that the clustering formation to be distinct, there can be cases with indistinct separation of data. Such cases can arise when the total number of targets under surveillance by a sensor is not constant during the period of analysis and when there are spurious data. Extention of this method to such scenarios should be investigated. Another area of research is the use of this method to improve the track maintenance at the sensor level. A disadvantage of this method is the computational load in the repeated applications of the batch process for intersensor data matching. However, an important feature of this approach is that any improvements to clustering procedures, correlation methods, or estimation techniques can be easily absorbed since it is modular.

Appendix A: Dynamics of Target Motion in Space

The model for the target motion in space is described by

$$\ddot{\bar{r}} = \frac{\mu}{r^3}\bar{r} + \bar{F}_G + \bar{F}_D + \bar{W}_1 \tag{A1}$$

where

$\bar{F}_G = H\bar{F}_B$
H = transformation matrix from the geocentric body-fixed system to an inertial system
$\bar{F}_B$ = gravitational acceleration in the body-fixed coordinate system
$\bar{F}_B = \nabla U$
∇ = the del operator

$$U = \frac{\mu}{r}\sum_{\ell=2}^{n}\sum_{m=0}^{\ell}\left(\frac{a_e}{r}\right)^{\ell} P_{\ell m}\,(\sin_\mu\phi)\,(C_{\ell m}\sin_\mu m\lambda + S_{\ell m}\sin_\mu m\lambda) \tag{A2}$$

where

μ = gravitational constant for Earth
λ = longitude
ϕ = geocentric latitude
a_e = mean radius of the Earth
$C_{\ell m}$, $S_{\ell m}$ = geopotential coefficients
$P_{\ell m}$ = Legendre polynomial of degree 1 and order m
$\bar{F}_D$ = drag acceleration vector
$\bar{W}_1$ = a vector of zero-mean white noise process with a power spectral density Q_1

Note that the second harmonic J_2 term in the geopotential is obtained with $\ell = 2$ and $m = 0$ in the equation for U. The term W_1 is added in Eq. (A1) to account for unmodeled forces.

Appendix B: Observation-State Relationships

1) Relative range:

$$R = \sqrt{(x_T - x_s)^2 + (y_T - y_s)^2 + (Z_T - Z_s)^2} + v_1 \tag{B1}$$

where x_T, y_T, Z_T = position states of the target, and x_s, y_s, z_s = position states of the sensor.

2) Relative range rate:

$$\dot{R} = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{R} + v_2 \tag{B2}$$

where x,y,z = relative position states, and $\dot{x},\dot{y},\dot{z}$ = relative velocity states,

3) Azimuth:

$$A_z = \tan^{-1}\left(\frac{y}{x}\right) + v_3 \tag{B3}$$

4) Elevation:

$$E\ell = \tan^{-1}\frac{z}{\sqrt{(x^2 + y^2)}} + v_4 \tag{B4}$$

The v_1–v_4 are Gaussian zero-mean white sequences. The associated variances are presented in Table 3.

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